Uncertainty Model for Quality Control of Stationary ADCP Measurement

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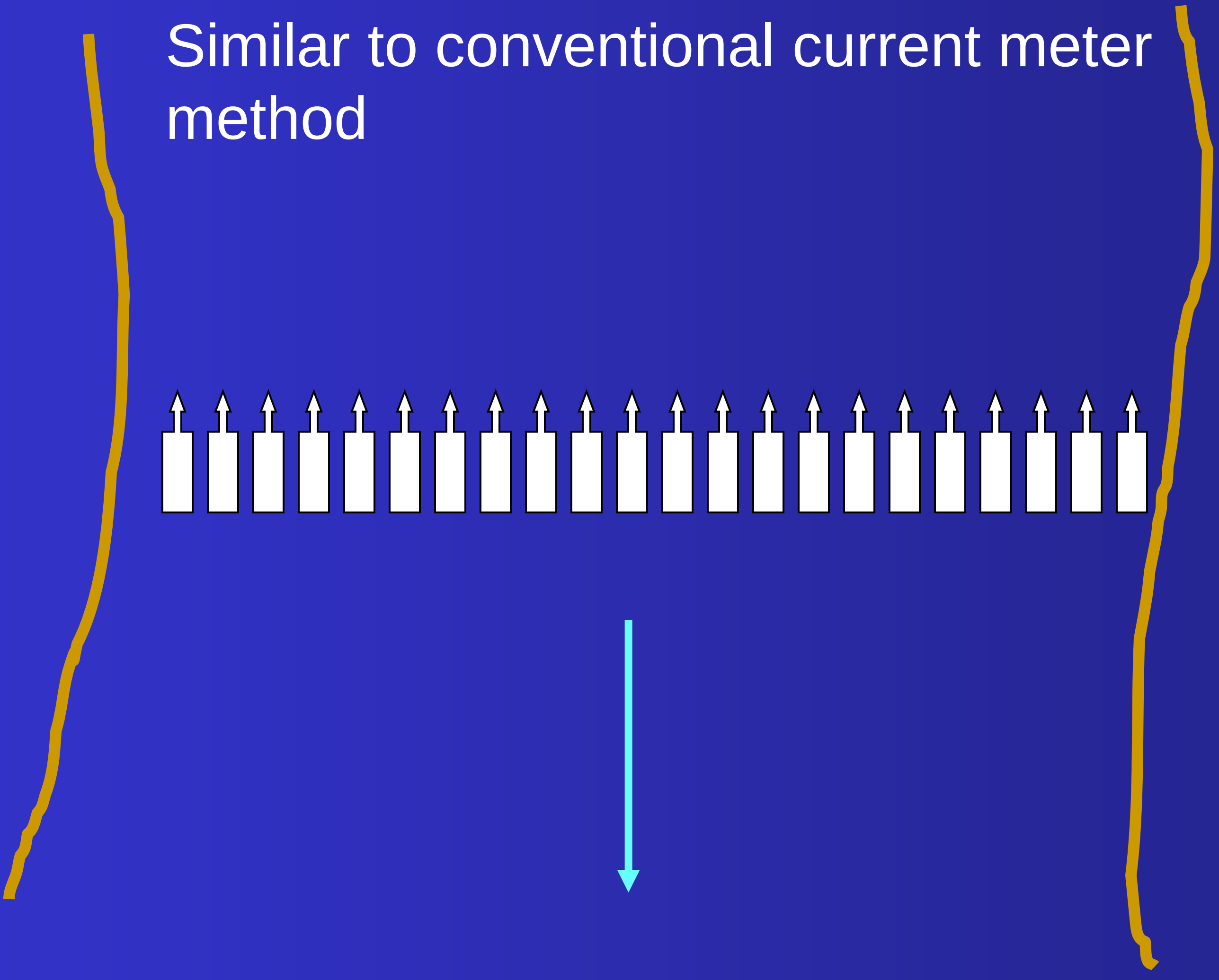
Stationary ADCP Discharge Measurement

Similar to conventional current meter method



Applications：Under Ice, Moving Bottom

Stationary ADCP: only to make a single measurement

Are we confident about the quality of a single measurement?

So, need uncertainty analysis.
Then how?

Development of Uncertainty Model

- Similar to the current meter method, the cross-section is divided into m sub-sections.
- Channel discharge Q is the sum of the discharge q_i of all subsections:

$$Q = F_s \sum_{i=1}^{m} b_i \, d_i \, V_i = F_s \sum_{i=1}^{m} q_i$$

Subsection q_i is the mean of ensemble $\boldsymbol{q_k}$

$$q_i = (\overline{q})_i = (\frac{1}{N}\sum_{k=1}^{N} q_k)_i$$

Ensemble q_k is the sum of top, middle, and bottom discharges

$$q_k = (q_{top} + q_{mid} + q_{btm})_k$$

Based on the uncertainty propagation principle, the standard uncertainty (relative) for single measurement is derived as:

$$u_Q = \sqrt{u_m^2 + u_{cal}^2 + \frac{u_b^2}{Q^2}\sum_{i=1}^{m} q_i^2 + \frac{1}{Q^2}\sum_{i=1}^{m}\delta_{qi}^2 + \frac{1}{Q^2}2\sum_{i=1}^{m-1}\delta_{qi}\,\delta_{qi+1}\,r_{i,i+1}}$$

Expanded uncertainty at 95% confidence level：

$$U_{95} = 2\,u_Q$$

Classification of Uncertainty

Traditional classification:

- Random uncertainty
- Systematic uncertainty

New classification (ISO 748 or ISO 5168):

- Type A：obtained from present data
- Type B：obtained from historical data or calibration

Uncertainty Components

Overall Uncertainty:

$$u_Q = \sqrt{u_A^2 + u_B^2}$$

Type A Uncertainty:

$$u_A = \frac{1}{Q}\sqrt{\sum_{i=1}^{m}\delta_{qi}^2 + 2\sum_{i=1}^{m-1}\delta_{qi}\,\delta_{qi+1}\,r_{i,i+1}}$$

Type B Uncertainty:

$$u_B = \sqrt{u_m^2 + u_{cal}^2 + \frac{u_b^2}{Q^2}\sum_{i=1}^{m}q_i^2}$$

Standard uncertainty of sub-section q (Type A):

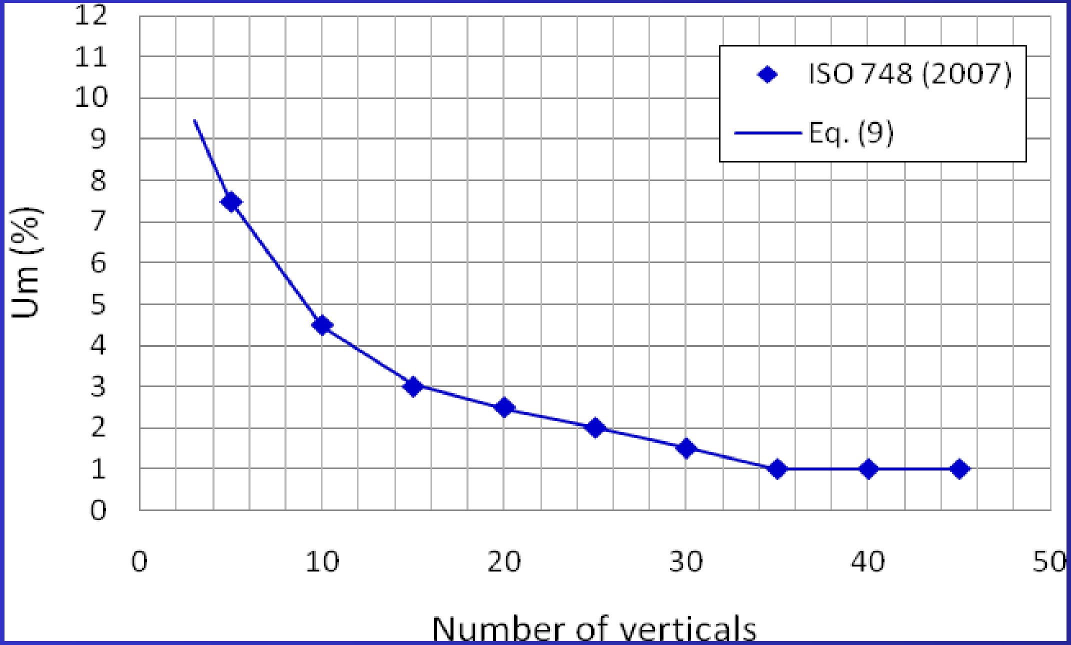
$$\delta_{qi} = \sqrt{\frac{\sum_{k=1}^{N} (q_k - \bar{q})_i^{\,2}}{N(N-1)}}$$

Correlation of adjacent sub-sections (Type A):

$$r_{i,i+1} = \frac{\sum_{k=1}^{N} (q_k - \bar{q})_i (q_k - \bar{q})_{i+1}}{\sqrt{\sum_{k=1}^{N} (q_k - \bar{q})_i^2}\sqrt{\sum_{k=1}^{N} (q_k - \bar{q})_{i+1}^2}}$$

Uncertainty due to Limited Number of Verticals (Type B) u_m obtained from ISO 748 (2007) Table E.6 data regression:

$$u_m \begin{cases} = [13.4286 - 1.5678\ m' + 0.0875\ m'^2 - 2.2525 \times 10^{-3} m'^3 + 2.1212\ \times 10^{-5} m'^4\ ]\% \\ \\ = 1\% \end{cases}$$



Calibration Uncertainty (Type B) u_{cal} According to ISO 748 (2007) or ISO 5168 (2005):

$$u_{cal} = \sqrt{u_{cm}^2 + u_{bm}^2 + u_{ds}^2}$$

U_{cm}、u_{bm}、u_{ds} = calibration uncertainty of current meter, width measurement, and depth measurement, respectively

U_{cal} is about 1%

The uncertainty model is applicable to the middle-section method and the mean-section method

Validation with five data sets

Data set	Site	Test date	Number of measurements
1	San Diego River	1/29/10	2
2	San Diego River	2/10/10	2
3	A stream in Canada (under ice measurement)	2/11/10	2
4	A Irrigation canal in California	4/30/10	4
5	San Diego River	1/4/11	8

Table 3. A comparison of the field RSU with the mean of the model estimated RSU (u_Q)

Comparison of Model Uncertainty and Field Uncertainty

Data set	Mean discharge (m^3/s)	Sample standard deviation (m^3/s)	Field uncertainty (%)	Model uncertainty (%)
1	2.086	0.006	0.34	3.12
2	11.02	0.240	2.73	2.71
3	996.98	8.577	1.08	2.30
4	16.37	0.092	0.61	2.03
5	4.105	0.069	1.75	3.57

Parameters affecting Stationary ADCP Discharge Measurement Quality

Parameter	Parameter Change	Uncertainty Change
Number of verticals	↑	↓
Measurement duration	↑	↓
ADCP performance	↑	↓
Turbulence intensity	↑	↑

Summary

- Combination of Type A and Type B uncertainties, complying with ISO uncertainty methodology
- Directly using ensemble discharge data to obtain Type A uncertainty components
- Validated by available field data: robust and reliable results
- Applicable to the mean-section method and the middle-section method
- Applicable to any ADCPs
- Built-into SxS Pro software

Suggestion

- Stationary ADCP discharge measurement quality control criterion:

$U \leq$ MPU (maximum permissible uncertainty)

MPU to be determined by a hydrology survey authority

Reference：Huang Hening (2012) “Uncertainty model for in situ quality control of stationary ADCP open-channel discharge measurement,” J. Hydraulic Engineering, ASCE, 138(1), 4-12.

Thank you !